Kinematics of a Free - Piston Expander

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Kholod. Tekhn. i Tekhnol., No. 18, 1974, TEKHNIKA, Kiev, pp. 28-33

A method of analyzing and optimizing a theoretical indicator diagram for a free-piston Longsworth expander /5/ (Fig. 1) was proposed in /2/.

The question of how to assure the selected form of the indicator diagram for a machine without having the kinematic coupling between the motion and gas distribution mechanisms arises in developing such an expander.

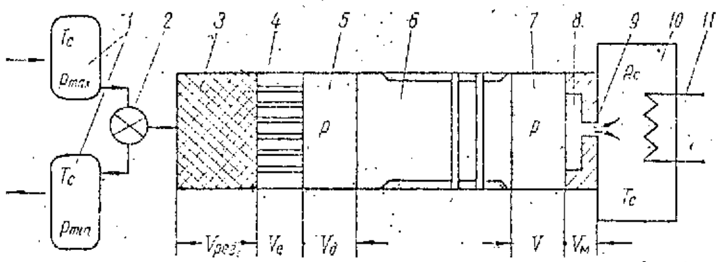

Fig. 1. Diagram of a Longsworth expander principle: 1 - Receivers; 2 - constraining valves; 3 - regenerator; 4 - refrigerator; 5 - expander working cavity; 6 - piston; 7 - damper working cavity; 8 - dead volume of the damper cavity; 9 - choke hole; 10 - buffer capacity; 11 - heat exchanger

The problem can be solved in a first approximation on the basis of the assumption that the friction and inertia forces are negligible compared to the pressure forces. In this case the piston velocity and duration of each of the processes are determined just by the gas discharge through the choke hole and the valves. The analytical dependences connecting the instantaneous values of the discharges to the thermodynamic and structural characteristics of the expander permit computation of the valve cyclogram.

In general form, the duration of any of the indicator diagram processes can be represented as

$$\tau_{n,\,n+1} = \int_{n}^{n+1} \frac{dM}{G} \qquad (1)$$

where τ is the time, M the mass of gas in the (constant or variable) volume under consideration, n, $n+1$ the numbering of the nodal points of indicator diagram (Fig. 2), G the instantaneous value of the discharge through the boundary of this volume. The discharge for the case of escape through the hole can be determined by means of the known relationships /1,4/:

$$G = f\mu \sqrt{\frac{2k}{k-1}\,\frac{(p')^2}{RT'}\left[\left(\frac{p''}{p'}\right)^{\frac{2}{k}} - \left(\frac{p''}{p'}\right)^{\frac{k+1}{k}}\right]} \qquad (2)$$

for the subcritical escape mode, and

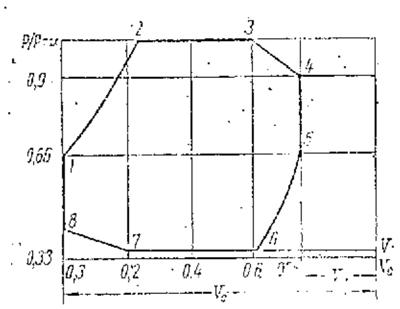

Fig. 2. Theoretical indicator diagram of the expander cavity: 1-2 - intake at elevated pressure; 2-3 - filling; 4-5 - exhaust for a fixed piston; 5-6 - exhaust with expulsion; 6-7 - expulsion; 7-8 - exhaust cutoff; 8-1 - pressure rise in the dead volume for a fixed piston

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(3) $$G_{\text{кр}} = f\mu \sqrt{\frac{2k}{k+1}\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}}\frac{(p')^2}{RT}} \qquad (3)$$

for supercritical escape.

Here p and T are the pressure and temperature, respectively, k is the adiabatic index, f the hole cross-sectional area, and μ the discharge coefficient.

In this case, the duration of the processes 2 - 3 - 4 - 5 depends on the tempo of escape from the damper cavity, assumed adiabatic, into the buffer capacity at the practically constant medium pressure p_c and temperature T_c, i.e.,

$$p' = p\ ;\quad p'' = p_c\ ;\quad T' = T(p)$$

where the single prime denotes quantities referring to the cavity out of which the gas flows and the double prime to the cavity into which the gas flows.

Hence, the discharge in the proces 2-3-4-5 is a function of one variable (the pressure p) in the damper cavity $G = f(p)$.

The time of the processes 6-7-8-1 is determined by the discharge from the buffer capacity into the damper cavity

$$p'' = p\ ,\quad p' = p_c\ ;\quad T' = T_c$$

and $G = f(p)$ as in the previous case.

The gas goes into the expander working volume or out of it through a valve an order of magnitude greater than the choke in processes 1-2 and 5-6 ; the pressure varies rapidly from p_1 to p_{max} or from p_5 to p_{min}. It is natural to assume that in this case a negligibly small quantity of gas succeeds in passing through the choke and the time $\tau_{1,2}$ and $\tau_{5,6}$ is determined by the discharge through the values, i.e., the constants $p' = p_{max}$, $T' = T_c$ and the variable $p'' = p_d$ should be substituted into (2) for the process 1-2 , and the constant $p'' = p_{min}$ and the variables $p' = p_{min}$ and $T' = T(p_d)$ for the process 5-6 .

Here p_{max} and p_{min} are the pressure in the receivers, and p_d is the moving value of the pressure in the expander cavity.

Let us note that under the assumption that the friction and inertia force is zero, the pressures p_d and p on the sides of the piston are equal, therefore, the discharge $G_{\text{кл}}$ through the valves in the processes 1-2 and 5-6 is also a function of the pressure in the damper cavity: $G_{\text{кл}} = f_1(p)$.

On the basis of the assumption that the damper cavity is adiabatic, we can write for either of the processes 2-3-4-5 and 6-7-8-1

(4)
$$dM = d\left(\frac{pV}{RT}\right) = d\left[p\,\frac{V_c - V_д(p)}{RT_n}\left(\frac{p_n}{T}\right)^{\frac{R-1}{k}}\right], \tag{4}$$

where p, V, T are the pressure, volume, and temperature of the damper volume, respectively, V_d is the volume of the expander cavity, V_o is the maximum volume of the damper cavity (taking account of its dead volume, p_n and T_n are the initial gas parameters for this process.

The mass increment element in the thermodynamic dead volume-expander cavity system in the processes 1-2 and 5-6 is

(5)
$$dM = d\left[p\,\frac{V_p + V_{per}}{RT_0} + p\,\frac{V_д(p)}{RT_n}\left(\frac{p_n}{p}\right)^{\frac{k+1}{k}}\right], \tag{5}$$

where V_p and V_{per} are the refrigerator and regenerator gas volumes, $T_o = \dfrac{V_{per} + V_p}{\dfrac{V_{per}}{T_{per}} + \dfrac{V_p}{T_p}}$ is the reduced temperature of the dead volume, T_p is the gas temperature in the refrigerator and T_{per} is the mean gas temperature in the regenerator (assumed constant).

The equation connecting the gas volume and pressure in the expander cavity $V_d(p)$ must be substituted into (4) and (5).

Therefore, (1) reduces to

(6)
$$\tau_{n,\,n+1} = \int_n^{n+1} \frac{dM}{G} = \int_n^{n+1} \frac{f(p)}{G(p)}\,dp, \tag{6}$$

where $f(p) = dM$.

After appropriate manipulations, (6) becomes for the separate processes

(7)
$$\tau_{1,2} = \int_1^2 C_1 \frac{dp}{p^{\frac{1}{k}}\sqrt{a - bp^{\frac{k-1}{k}}}}; \tag{7}$$

(8)
$$\tau_{3,4} = \int_3^4 C_3 \frac{dp}{p^{\frac{k-1}{k}}\sqrt{bp^{\frac{k-1}{k}} - a}}; \tag{8}$$

(9)
$$\tau_{4,5} = \int_4^5 C_4 \frac{dp}{p^{\frac{k-1}{k}}\sqrt{bp^{\frac{k-1}{k}} - a}}; \tag{9}$$

$$\tau_{5,6} = I_1 + I_2 + I_3,$$

where

(10)
$$I_1 = \int_5^6 C_5 \frac{dp}{p^{\frac{k-1}{k}}\sqrt{a - bp^{\frac{1-k}{k}}}}; \tag{10}$$

(11)
$$I_2 = \int_5^6 C_5 \frac{dp}{p^{\frac{1-2k}{k}}\sqrt{a - bp^{\frac{1-k}{k}}}}; \tag{11}$$

(12)
$$I_3 = \int_5^6 C_5 \frac{dp}{p^{\frac{2k-1}{k}}\sqrt{a - bp^{\frac{1-k}{k}}}}; \tag{12}$$

(13)
$$\tau_{7,8} = \int_7^8 C_7 \frac{c_1 + c_2 p^{\frac{k-1}{k}}}{p^{\frac{1}{k}} \sqrt{a - bp^{\frac{k-1}{k}}}} dp; \tag{13}$$

(14)
$$\tau_{8,1} = \int_8^1 C_5 \frac{dp}{p^{\frac{1}{k}} \sqrt{a - bp^{\frac{k-1}{k}}}}, \tag{14}$$

and C, a, b are constant coefficients for a given process.

The problem of determining the emptying or filling time for the capacity in the subcritical escape mode is usually solved by numerical methods /1, 4/.

In this case it admits of an analytical solution. The quadratures can be found by using the substitution $x = p^{\frac{k-1}{k}}$ for the integrals (7), (13) and (14), $x = p^{\frac{2-k}{k}}$ for (11), and $x = p^{\frac{1-k}{k}}$ for (12). By substituting $x = p^{\frac{1}{k}}$ the integrands of (8) and (9) are reduced to

(15)
$$d\tau = C\,(bx^{k-1} - a)^{-\frac{1}{2}}\,dx. \tag{15}$$

For a monotomic gas, helium, say, $k = 5/3$, and the binomial differential (15) is rationalized by the Chebyshev method by the substitution

$$bx^{-2/3} - a = z^2$$

after which integration is not difficult.

The expression (10) is integrated by an analogous method.

Since the pressure is constant in the processes 2-3 and 6-7, they are described by algebraic equations.

If the ratio between the pressures governing the escape mode passes through the critical value, (1) should be represented as

(16)
$$\tau_{n,n+1} = \int_n^m \frac{dM}{G_{\text{кр}}} + \int_m^{n+1} \frac{dM}{G}, \tag{16}$$

where m is the point of the process corresponding to the critical pressure.

The initial values for each of the processes are computed by means of the condition of periodicity of the expander working cycle.

The method described permitted representation of the operating cycle time as

(17)
$$\tau_{\text{ц}} = \sum \tau_{n,n+1} = (FL + V_m) \frac{1}{f\mu \sqrt{RT_c}} (\Phi_{2,3} + \Phi_{3,4} + \Phi_{4,5} + \Phi_{6,7} + \Phi_{7,8} + \\ + \Phi_{8,1}) + (FL + V_m) \frac{1}{f\mu_{\text{к}}} \left(\frac{1}{T_p \sqrt{\frac{R}{T_c}}} \Phi_{2,3} + \frac{1}{\sqrt{RT_0}} \Phi_{5,6} \right), \tag{17}$$

where f and f_k are the passage section areas of the choke and valve, respectively, μ and $\mu_{\text{к}}$ are the mean values of the choke and valve discharge coefficients, in the pressure range under consideration, respectively, V_m is the dead volume of the damper cavity F and L are the piston stroke and area, and $\Phi_{n,\,n+1}$ are dimensionless complexes comprised of the parameters characterizing the structural peculiarities of the expander and its operating mode:

$$\Phi_{n,\,n+1} = f\left(T_0,\ T_{п},\ \frac{V_{per}}{V_c},\ \frac{V_p}{V_c},\ \frac{V_3}{V_c},\ \frac{V_7}{V_c},\ \frac{V_m}{V_c},\ \frac{p_1}{p_{max}},\ \frac{p_5}{p_{max}},\ \frac{p_{min}}{p_{max}},\ \frac{p_c}{p_{max}},\ \frac{p_{min}}{p_c}\right) \tag{18}$$

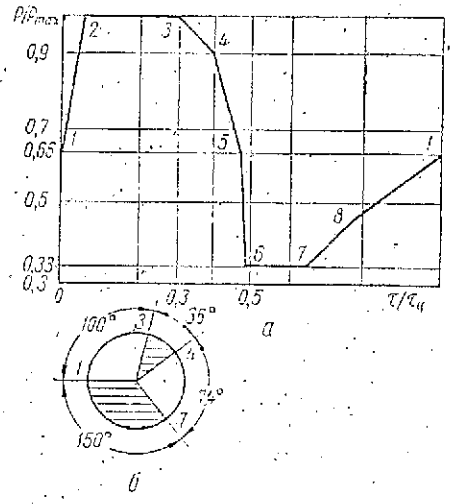

Fig. 3. Graphs of the pressure change throughout the operating cycle (a) and valve cyclogram (b).

The expression (16) permits computation of the valve cyclogram which will assure the given shape of the indicator diagram.

As an illustration, an expander was computed with the following characteristics:

$$F = 0{,}03\ м;\ L = 0{,}03\ м;\ \frac{V_{per}}{V_c} = 1{,}4;\ \frac{V_p}{V_c} = 0{,}25;\ \frac{V_m}{V_c} = 0{,}25;\ \frac{V_3}{V_c} = 0{,}61;$$

$$\frac{V_7}{V_c} = 0{,}2;\ \frac{p_{max}}{p_{min}} = 3;\ \frac{p_c}{p_{min}} = 2;\ \frac{p_{max}}{p_1} = \frac{p_{max}}{p_5} = \frac{p_{max}}{p_1} = 1{,}5;\ T_p = 150^\circ\ K;$$

$\mu f = 1 \cdot 10^{-4}$, $\mu_к f_к = 40 \cdot 10^{-6}\ м^2$ for $T_c = 300^\circ$ K.

The theoretical indicator diagram of such an expander is shown in Fig. 2, and the corresponding valve cyclogram in Fig. 3.

Let us note that the cycle duration is related practically linearly to the choke hole cross-sectional area.

Therefore, a proportional change in the number of valve drive rotations and in the choke cross-section permits regulating the frequency of the working process without essential distortion of the indicator diagram.

Jan. 20, 1973

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